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How to Power Quadrupole 1VQ5 Asymmetrically?

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Abstract: Algorithm is derived on how to power the quadrupole 1VQ5 asymmetrically in order to achieve horizontal and vertical steering components in addition to the quadrupole component, and to allow any component independently adjustable without affecting the others.

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1 Requirements

The beam line BL1U [1], planned for the UCN project, has been laid out such that the existing two steering magnets 1VSM2 (vertical steering) & 1VSM3 (horizontal steering) will be eliminated out of the beam line; as a compensation the quadrupole 1VQ5 (horizontal defocusing) will be rewired with additional four power supplies connected to the poles to accomplish the function of these two steering magnets. **Each of the four trim power supplies has to be wired in a way that it runs current in the same direction as the main current though each pole. Otherwise, the trim power supplies would have to absorb current from the main power supply, which is not allowed to happen.** This is a constraint imposed by the power supply.

To that end, a question was raised: how to power the magnet properly such that it will induce steering components in both horizontal and vertical directions, also the horizontal steering component, the vertical steering component and the net quadrupole component are independently adjustable?

Algorithm is derived in this note to answer this question.

2 Algorithm

As is shown in the diagram Fig.1, the 4-pole's current is represented as

$$A = \begin{bmatrix} +I + I_1 & -I - I_4 \\ & \\ -I - I_2 & +I + I_3 \end{bmatrix}, \tag{1}$$

where I denotes the main current from the big power supply ($I > 0$), I_k (k=1,2,3,4) denotes the small current from each trim power supply and must be ≥ 0 for all $k's$; the $+$ sign means that the current is flowing in a direction identical to that on the North pole while the $-$ sign means the current is flowing as on the South pole.

Eq.(1) has to decompose into 3 terms (superposition principle), namely, quadrupole component, vertical steering component and horizontal steering component:

$$A = \begin{bmatrix} +Q & -Q \\ -Q & +Q \end{bmatrix} + \begin{bmatrix} -V & +V \\ -V & +V \end{bmatrix} + \begin{bmatrix} -H & -H \\ +H & +H \end{bmatrix}, \tag{2}$$

where **Q, $|V|$ and $|H|$ denote certain amount of currents. Again, on any pole, a positive current value means this is North pole, while a negative current value means South pole. Note that Q must be > 0 because this is a horizontal defocusing quad as required, V and H can be positive value or negative value or zero.**

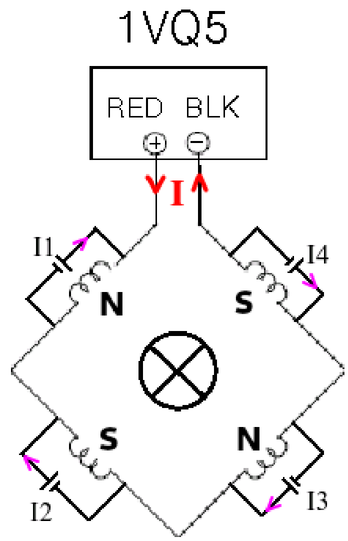

Figure 1: *Diagram showing that the four trim power supplies are wired to run current in the same direction as the main current through each pole. Note that the pole's polarities are labelled in such a way that the proton beam is moving into the paper and the 1VQ5 is a horizontally defocusing quadrupole magnet as required.*

Here the beam is moving into the paper, so when $V > 0$, it steers beam (protons) upward; when $H > 0$, it steers beam to the right (looking downstream).

Eq.(1) and (2) become

$$\begin{aligned} -I - I_4 &= -Q + V - H\,, \\ +I + I_1 &= +Q - V - H\,, \\ -I - I_2 &= -Q - V + H\,, \\ +I + I_3 &= +Q + V + H\,. \end{aligned} \tag{3}$$

We thus obtain 4 equations (i.e. constraints) for the 3 effective (i.e. virtual) current values Q, V, H as well as for I_3

$$\begin{aligned} Q &= I + (I_4 + I_2)/4 + (I_3 + I_1)/4\,, \\ V &= (I_2 - I_1)/2\,, \\ H &= (I_4 - I_1)/2\,, \\ I_3 &= I_4 + I_2 - I_1\,, \end{aligned} \tag{4}$$

in which we have only 4 instead of 5 independent physical variables (i.e. I and $I_1\,, I_2\,, I_4$) for the power supplies. I_3 is NOT an independently adjustable parameter. Instead, I_3 is bounded by the other three.

Eq.(4) can be rewritten to represent the relationship between the actual/physical

currents and the effective/virtual currents plus the physical I_1

$$
\begin{aligned}
I_2 &= 2V + I_1\,, \\
I_3 &= 2H + 2V + I_1\,, \\
I_4 &= 2H + I_1\,, \\
I &= Q - H - V - I_1\,.
\end{aligned} \tag{5}
$$

Keep in mind that in these equations, numerically H and V can be of any sign or equal to zero, while $I_k\,(k = 1, 2, 3, 4)$ and I as well as Q must be constantly ≥ 0 at any time of operation.

Initial settings: In the 480 MeV production tune of BL1A since July 2010 till now, 1VQ5 has been running at ∼200.0 A, while SM2V running at about −1.0 A (at Normal Polarity), SM3H running at about +2.0 A (at Normal Polarity). Therefore, as initial settings for the five power supplies of 1VQ5, we shall begin with the following values:

$$I{=}200\,\mathrm{A}, \quad I_4 = I_1 = I_2 = I_3 = 0\,\mathrm{A}.$$

In this case, there is no extra steering components but only the quadrupole component induced. However, if it's needed, we can adjust any of the three effective/virtual components independently of the other two.

Case 1: if we want to adjust only the horizontal steering while keeping the vertical steering and quadrupole component unchanged, then we need

$$
\begin{aligned}
\Delta H = &\quad (\Delta I_4 - \Delta I_1)/2\,, \\
\Delta V = &\quad (\Delta I_2 - \Delta I_1)/2 = 0 \Longrightarrow \Delta I_2 = \Delta I_1\,, \\
\Delta Q = &\quad \Delta I + (\Delta I_4 + \Delta I_2)/2 = 0 \Longrightarrow \Delta I = -(\Delta I_4 + \Delta I_2)/2\,, \\
\Delta I_3 = &\quad \Delta I_4 + \Delta I_2 - \Delta I_1\ .
\end{aligned} \tag{6}
$$

where Δ means an amount of change to make, relative to the present setting. Clearly, the solution is not unique; there are multiple solutions. To minimize the adjustments, we shall choose the simplest one.

If we want to increase H to steer the beam to the right, i.e. $\Delta H > 0$, then we need $\Delta I_4 > \Delta I_1$. We choose

$$\Delta I_1 = 0\,,$$

Eq.(6) thus gives

$$\Delta I_4 = 2\Delta H\,, \quad \Delta I_2 = 0\,, \quad \Delta I_3 = 2\Delta H\,; \qquad \Delta I = -\Delta H\,.$$

Conversely, if we want to decrease H to steer the beam to the left, i.e. $\Delta H < 0$, then we need $\Delta I_4 < \Delta I_1$. We choose

$$\Delta I_4 = 0\,,$$

thus

$$\Delta I_1 = -2\Delta H\,,\ \ \Delta I_2 = -2\Delta H\,,\ \ \Delta I_3 = 0\,; \qquad \Delta I = \Delta H\,.$$

All these changes are $\Delta I_k \geq 0$ ($k = 1, 2, 3, 4$), thus ensure $I_k \geq 0$. This completely satisfies the requirement from the power supply (they are not allowed to reverse the polarities). But notice that $\Delta I < 0$. This is not a problem because $|\Delta I|$ in magnitude is expected to be $\leq 10\,\mathrm{A}$ while the initial I is as large as $200\,\mathrm{A}$. There is no chance to cause $I < 0$ at any time.

Case 2: if we want to adjust only the vertical steering while keeping the horizontal steering and quadrupole component unchanged, then we have

$$\begin{aligned}
\Delta V =& \ \ (\Delta I_2 - \Delta I_1)/2\,, \\
\Delta H =& \ \ (\Delta I_4 - \Delta I_1)/2 = 0 \Longrightarrow \Delta I_4 = \Delta I_1\,, \\
\Delta Q =& \ \ \Delta I + (\Delta I_4 + \Delta I_2)/2 = 0 \Longrightarrow \Delta I = -(\Delta I_4 + \Delta I_2)/2\,, \\
\Delta I_3 =& \ \ \Delta I_4 + \Delta I_2 - \Delta I_1\ \ .
\end{aligned} \tag{7}$$

Similar to the Case 1, if we want to increase V to steer the beam upward, i.e. $\Delta V > 0$, then we need $\Delta I_2 > \Delta I_1$. We choose

$$\Delta I_1 = 0\,,$$

thus

$$\Delta I_4 = 0\,,\ \ \Delta I_2 = 2\Delta V\,,\ \ \Delta I_3 = 2\Delta V\,; \qquad \Delta I = -\Delta V\,.$$

Conversely, if we want to decrease V to steer the beam downward, i.e. $\Delta V < 0$, then we need $\Delta I_2 < \Delta I_1$. We choose

$$\Delta I_2 = 0\,,$$

thus

$$\Delta I_4 = -2\Delta V\,,\ \ \Delta I_1 = -2\Delta V\,,\ \ \Delta I_3 = 0\,; \qquad \Delta I = \Delta V\,.$$

As well, all these changes ensure $I_k \geq 0$ as required.

Case 3: if we want to adjust only the quadrupole component while keeping the horizontal and vertical steering components unchanged, we have

$$\begin{aligned}
\Delta Q =& \ \ \Delta I + (\Delta I_4 + \Delta I_2)/2\,, \\
\Delta H =& \ \ (\Delta I_4 - \Delta I_1)/2 = 0 \Longrightarrow \Delta I_4 = \Delta I_1\,, \\
\Delta V =& \ \ (\Delta I_2 - \Delta I_1)/2 = 0 \Longrightarrow \Delta I_2 = \Delta I_1\,,
\end{aligned} \tag{8}$$

We choose

$$\Delta I_4 = 0\,,$$

so

$$\Delta I_1 = 0\,,\ \ \Delta I_2 = 0\,,\ \ \Delta I_3 = 0\,; \qquad \Delta I = \Delta Q\,.$$

This means that to achieve the goal we shall only need to adjust the main current I, i.e. **increase or decrease** it by an amount ΔI.

However, it should be aware that when the beam centroid is misaligned from the quadrupole's axis, the above adjustment made only with ΔI will steer the beam. To remedy this situation, we look at one more case next.

Case 4: we want to adjust only the quadrupole component while keeping the beam steering unchanged even if the beam is off-centre. This is to say that we need to vary the field gradient while maintaining the field value (B_x and B_y both) at the beam position. We thus need to satisfy the following two conditions

$$\begin{aligned} \Delta V/\sqrt{2} &= \quad \Delta Q\ y/a\,, \\ \Delta H/\sqrt{2} &= \quad \Delta Q\ x/a\,, \end{aligned} \tag{9}$$

where a denotes the quad's aperture radius, x and y denote the beam centroid displacement from the axis. The factor $\sqrt{2}$ comes from that the steerer's pole gap is about equal to $\sqrt{2}\ a$.

Inserting

$$\begin{aligned} \Delta V &= \quad (\Delta I_2 - \Delta I_1)/2\,, \\ \Delta H &= \quad (\Delta I_4 - \Delta I_1)/2\,, \end{aligned} \tag{10}$$

into Eq.(9), we obtain

$$\begin{aligned} \Delta I_2 - \Delta I_1 &= \quad 2\sqrt{2}\ \Delta Q\ y/a\,, \\ \Delta I_4 - \Delta I_1 &= \quad 2\sqrt{2}\ \Delta Q\ x/a\,. \end{aligned} \tag{11}$$

We have another two equations

$$\begin{aligned} \Delta I &= \quad \Delta Q - (\Delta I_4 + \Delta I_2)/2\,, \\ \Delta I_3 &= \quad \Delta I_4 + \Delta I_2 - \Delta I_1 \ \ . \end{aligned} \tag{12}$$

With 5 unknowns in 4 equations, the solution is not unique. Still, we shall choose the simplest one. Since x, y can be either ≥ 0 or < 0, to ensure $\Delta I_k \geq 0$ as was stated above, we end up getting various solutions depending on the signs of x, y and ΔQ.

If $x\Delta Q \geq 0$ and $y\Delta Q \geq 0$, then we choose

$$\Delta I_1 = 0\,, \tag{13}$$

so we get

$$\begin{aligned} \Delta I_4 = &\ 2\sqrt{2}\ \Delta Q\ x/a\,, \\ \Delta I_2 = &\ 2\sqrt{2}\ \Delta Q\ y/a\,, \\ \Delta I_3 = &\ 2\sqrt{2}\ \Delta Q\ (x+y)/a\,, \\ \Delta I = &\ \Delta Q[1-\sqrt{2}\ (x+y)/a]\,. \end{aligned} \tag{14}$$

If $x\Delta Q \leq 0$ and $y\Delta Q \leq 0$, then we choose

$$\Delta I_1 = -2\sqrt{2}\ \Delta Q\ (x+y)/a\,, \tag{15}$$

so we have

$$\begin{aligned} \Delta I_4 = &\ -2\sqrt{2}\ \Delta Q\ y/a\,, \\ \Delta I_2 = &\ -2\sqrt{2}\ \Delta Q\ x/a\,, \\ \Delta I_3 = &\ 0\,, \\ \Delta I = &\ \Delta Q[1+\sqrt{2}\ (x+y)/a]\,. \end{aligned} \tag{16}$$

If $x\Delta Q \geq 0$ and $y\Delta Q \leq 0$, then we choose

$$\Delta I_1 = -2\sqrt{2}\ \Delta Q\ y/a\,, \tag{17}$$

so we have

$$\begin{aligned} \Delta I_4 = &\ 2\sqrt{2}\ \Delta Q\ (x-y)/a\,, \\ \Delta I_2 = &\ 0\,, \\ \Delta I_3 = &\ 2\sqrt{2}\ \Delta Q\ x/a\,, \\ \Delta I = &\ \Delta Q[1-\sqrt{2}\ (x-y)/a]\,. \end{aligned} \tag{18}$$

If $x\Delta Q \leq 0$ and $y\Delta Q \geq 0$, then we choose

$$\Delta I_1 = -2\sqrt{2}\ \Delta Q\ x/a\,, \tag{19}$$

so we have

$$
\begin{aligned}
\Delta I_4 = &\ \ 0\,, \\
\Delta I_2 = &\ \ 2\sqrt{2}\ \Delta Q\ (y-x)/a\,, \\
\Delta I_3 = &\ \ 2\sqrt{2}\ \Delta Q\ y/a\,, \\
\Delta I = &\ \ \Delta Q[1-\sqrt{2}\ (y-x)/a]\,.
\end{aligned}
\tag{20}
$$

Lastly, we should point out that the beam displacement x and y needs to be determined beforehand. This can be measured by 'dithering' the quad: simply change the quad setting and look at the steering effect at monitor downstream.

3 Discussions

As mentioned above, in Eq.(4) there are 4 constraints for the 3 effective current values Q, V, H and for I_3

$$
\begin{aligned}
Q &= I + (I_4+I_2)/4 + (I_3+I_1)/4\,, \\
V &= (I_2-I_1)/2\,, \\
H &= (I_4-I_1)/2\,, \\
I_3 &= I_4+I_2-I_1\,.
\end{aligned}
\tag{21}
$$

While there are 5 power supplies. Seemingly, this implies that one of the trim power supplies is superfluous. But this is not true because only 3 of the trim supplies are independently adjustable, while the other one is bounded by the last equation in Eq.(21).

For example, we assume that I_3 is superfluous and eliminate it. This is equivalent to simply setting $I_3 = 0$ in Eq.(21). So the last equation becomes

$$
I_1 = I_4 + I_2\,. \tag{22}
$$

And then, if we want to increase H only (i.e. $\Delta H > 0$), while keeping V and Q both unchanged (i.e. $\Delta V = 0 = \Delta Q$), then we need $\Delta I_4 > \Delta I_1$. We choose

$$
\Delta I_1 = 0\,,
$$

so we get

$$
\Delta I_4 = 2\Delta H\,,\ \ \Delta I_2 = 0 \qquad (\text{and}\ \ \Delta I = -\Delta H\,).
$$

So $\Delta I_4 + \Delta I_2 = 2\Delta H > 0 = \Delta I_1$. These do not meet the constraint of $\Delta I_4 + \Delta I_2 = \Delta I_1$ indicated by Eq.(22).

In actual applications, one of the concerns with the asymmetrical quad is that any of the 4 trim power supplies could have run up and exceeded its full scale. This could happen at any step of the operation, as the trim supplies are much smaller than the main supply. Whenever this occurs, this supply (or these supplies) should be set to their full scale(s), and the other ones should be adjusted accordingly to keep the effective component(s) unchanged or minimally changed relative to their desired settings.

Also, it could happen that all the 4 trim supplies have run up very high, approaching but not hitting their full scale(s) yet. In order to avoid this occurrence, **in the end of every operation we should proceed to decrease all the 4 trim supplies' settings by an amount that is equal to the lowest setting of the 4 supplies at present, while increase the main supply's setting by the same amount.** This way keeps all the 3 effective components unchanged, as indicated by Eq.(21).

Also, **should be pointed out that with 4 trim power supplies configuration it allows to steer the beam to any of the 4 directions: up and down, left and right, which is equivalent to the capability of reversing the steerer's polarity. This is good. But, the in-quad steering fields in both h and v directions are not as uniform as those in a real isolated steering magnet because an air gap exists between the neighbouring poles. This implies that the horizontal steering field strength depends on particle's y-coordinate, while the vertical steering field strength depends on particle's x-coordinate**. These cause an error in the particle's deflections and thus an emittance growth, which is not god.

4 Acknowledgements

Comments and suggestions from R. Baartman are very much appreciated.

References

[1] Charles Davis, *BL1U, Sections 1 & 2*, Design Note TRI-DN-1306, Document-74308, Release 01, 2013-00-00.